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# Side Information in Coded Aperture Compressive Spectral Imaging

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# Side Information in Coded Aperture Compressive Spectral Imaging

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## ABSTRACT

Coded aperture compressive spectral imagers sense a three-dimensional cube by using two-dimensional projections of the coded and spectrally dispersed source. These imagers systems often rely on focal plane arrays (FPAs), spatial light modulators (SLMs), digital micromirror devices (DMDs), and dispersive elements. The use of the DMDs to implement the coded apertures facilitates the capture of multiple projections, each admitting a different coded aperture pattern. The DMD allows not only to collect the sufficient number of measurements for spectrally rich scenes or very detailed spatial scenes but to design the spatial structure of the coded apertures to maximize the information content on the compressive measurements. Although sparsity is the only signal characteristic usually assumed for reconstruction in compressing sensing, other forms of prior information such as side information have been included as a way to improve the quality of the reconstructions. This paper presents the coded aperture design in a compressive spectral imager with side information in the form of RGB true color images of the scene. The use of RGB images as side information of the compressive sensing architecture has two main advantages: the RGB is not only used to improve the reconstruction quality but to optimally design the coded apertures for the sensing process. The coded aperture design is based on the RGB scene and thus the coded aperture structure exploits key features such as scene edges. Real reconstructions of noisy compressed measurements demonstrate the benefit of the designed coded apertures in addition to the improvement in the reconstruction quality obtained by the use of side information.

**Keywords:** Coded aperture design, Side Information, Compressive sensing.

## 1. INTRODUCTION

Coded aperture compressive spectral imagers capture two-dimensional projections by multiplexing the spatio-spectral information of a scene through a coded aperture and a dispersive element.<sup>1</sup> These imagers systems often rely on focal plane arrays (FPAs), spatial light modulators (SLMs), digital micromirror devices (DMDs), and dispersive elements. The use of the DMDs to implement the coded apertures facilitates the capture of multiple projections, each admitting a different coded aperture pattern. The DMD allows not only to collect the sufficient number of measurements for spectrally rich scenes or very detailed spatial scenes but to design the spatial structure of the coded apertures to maximize the information content on the compressive measurements. The snapshots are measured by a FPA, which pixel pitch determines the spatial and spectral resolution of the reconstruction.<sup>2</sup> Algorithms to solve the quadratic formulation resulting from the ill-conditioned, linear systems of equations are then used to reconstruct the data cube, using representation basis where the undersampled signals admit sparse representations.

The use of prior information have been extensively studied in order to improve the reconstruction of those undersampled signals. For instance,<sup>3</sup> took advantage of the information about the support of the signal at the decoder for its reconstruction. In,<sup>4</sup> a side information (SI)-aided compressed sensing reconstruction is considered, it uses a noisy version of the same signal, obtained at the decoder to reconstruct the original signal through a SI-aided approximate message passing (SI-AMP) algorithm. However, the use of prior information as an additional

measurement known as side information has been recently considered to aid the reconstruction of signals, which are sparse in certain domains. In tomography imaging, previous scans of a subject used as side information enables accurate reconstruction of dynamic CT images.<sup>5</sup> The works in<sup>6</sup> and<sup>7</sup> reconstruct images using a side information snapshot, both of them use  $\ell_1$ -norm based minimization for recovery, by adding an additional term that accounts for the distance between the recovered image and the side information snapshot. Some approaches as<sup>8,9</sup> developed hybrid cameras to acquire simultaneously side RGB information in addition to multispectral imaging with low spatial resolution.

A novel approach is presented in this paper that optimally design the set of coded apertures to use in the acquisition of the compressive projections taking advantage of the DMD element. In essence, a side RGB image provides a-priori information to design the coded aperture in order to sense the scene in a structured format such that the spatial frequencies of the image are better reconstructed. In addition, the side RGB information is used to improve the reconstruction quality by its use during the reconstruction process. This is, the side RGB information is twice exploited, for the sensing process and the reconstruction.

In this paper we present two applications of the side information in compressive spectral imaging, to optimally design the set of coded apertures while improving the reconstruction quality of the spatio-spectral data cubes. Simulations and experimental results demonstrate the quality achieved by using this approach.

## 2. CASSI MODEL FOR SPECTRAL IMAGING

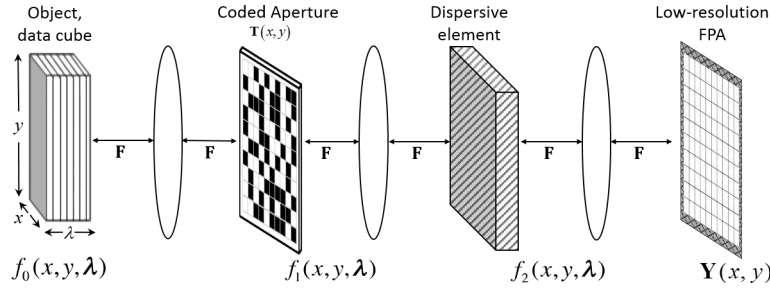


Figure 1. CASSI Model. The data cube is coded, spectrally dispersed and integrated on the FPA.

A well known coded aperture compressive spectral imager is the coded aperture snapshot spectral imager (CASSI). It captures multiplexed 2D projections of a spatio-spectral scene using a snapshot. The CASSI architecture is composed by a coded aperture, a dispersive element, and a focal plane array (FPA) as illustrated in Fig. 1. The source, a spatio-spectral image defined as  $(\mathbf{F}_k)_{mn}$ , where  $m$  and  $n$  index the spatial coordinates and  $k$  determines the  $k^{th}$  spectral band is coded by a discretized coded aperture  $\mathbf{T}_{mn}^\ell$ , where  $\ell$  index the number of snapshots to be captured, each one using a different coded aperture. Using this notation, the FPA measurement  $\mathbf{Y}_{mn}^\ell$  can be written as

$$\mathbf{Y}_{mn}^\ell = \sum_{k=0}^{L-1} \mathbf{T}_{m(n-k)}^\ell (\mathbf{F}_k)_{m(n-k)}, \quad (1)$$

where  $m, n = 0, 1, \dots, N-1$ ,  $k = 0, 1, \dots, L-1$ ,  $\ell = 0, 1, \dots, K-1$ , and the dispersion effect is modeled at the pixel level in the horizontal dimension in both the coded aperture and the source. Alternatively, the spectral signal can be expressed as  $\mathbf{F} \in \mathbb{R}^{N \times N \times L}$ , or its vector representation  $\mathbf{f} \in \mathbb{R}^{N \cdot N \cdot L}$ , which is  $S$ -sparse on some basis  $\Psi$ , such that  $\mathbf{f} = \Psi\theta$  can be approximated by a linear combination of  $S$  vectors from  $\Psi$  with  $S \ll (N \cdot N \cdot L)$ . Following this notation, the CASSI projections in Eq. 1 can be rewritten in the standard form of an undetermined system of linear equations

$$\mathbf{Y}^\ell = \mathbf{A}^\ell \theta = \mathbf{H}^\ell \Psi \theta + \omega \quad (2)$$

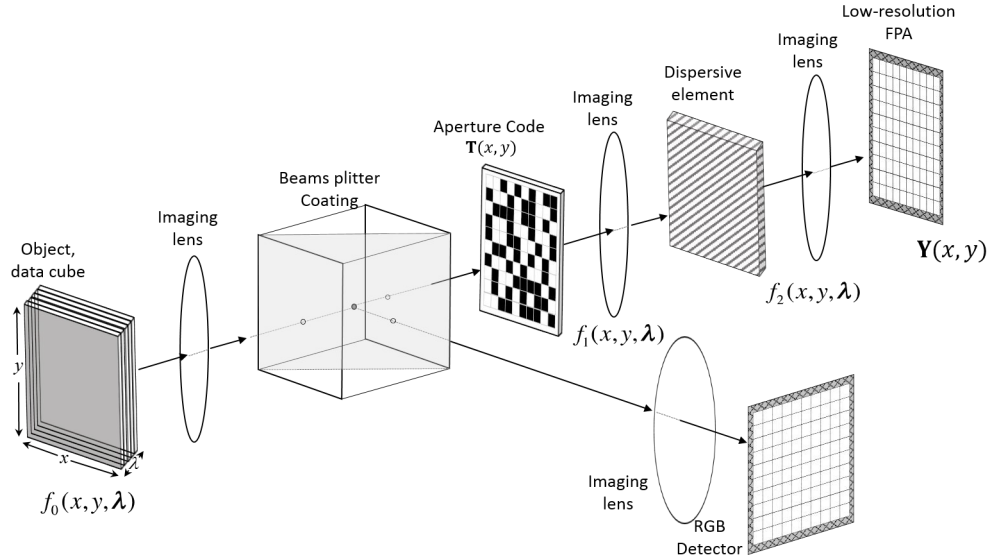


Figure 2. Schematic architecture of CASSI with RGB side information. A beam splitter is utilized to divide the incident light into two different cameras.

where  $\mathbf{H}^\ell$  is a structured matrix determined by the coded aperture entries and the dispersive element effect, the matrix  $\mathbf{A} = \mathbf{H}\Psi$  is the CASSI sensing matrix,  $\boldsymbol{\theta}$  is a  $S$ -sparse representation of the data cube in a 3-dimensional base  $\Psi$ , and  $\boldsymbol{\omega}$  represent the noise in the system.

### 3. COMPRESSIVE SPECTRAL IMAGING WITH SIDE INFORMATION

In spectral imaging, a RGB image of the same target can be captured in two distinct ways, one is to use different cameras in the same path of the spectral image and another way is to combine the two systems using a beam splitter. Depending on the application, any of those methods should be utilized.<sup>10</sup> An example of an architecture is presented in figure 2. This architecture combines a spectral system and a traditional RGB imaging system by the use of a beam splitter dividing the incident light into the two paths. Independently of the approach to acquire the RGB image, after registration of the images, the RGB image could be used as side information to aid the reconstruction. In addition, a novel approach is developed in this work, which take advantage of the side information also during the sensing process. We propose an edge-based structured coded aperture, which allows to improve the quality of the measurements in order to carefully sense the borders of the objects in the scene, so the RGB information is used to configure the DMD and can adapt itself to different scene content during acquisition.

#### 3.1 Coded aperture design based on side information

Traditional coded apertures in CASSI such as Hadamard matrices, S matrices, and Bernoulli random matrices do not exploit the information a-priori of the scene, when it is available. For instance the random patterns try to sense in a uniform way the scene having control only on the transmittance or the percentage of information allowed to be sensed. However, the scenes are not usually uniform, they contains intensity variations, intensity discontinuities in some directions and uniform/non-uniform patches. Therefore, the intuition behind is that the properly design of the coded aperture ensembles could improve the quality of the reconstruction.

The reconstructed spatial quality of spectral images is affected mainly in the sections containing numerous details. More specifically, the edges of the objects in the scene are poorly reconstructed using traditional coded apertures, such as random and hadamard coded apertures. Figure 3 presents the reconstruction of two bands of a spectral data cube obtained through simulations. For the simulation, the data cube is sensed by the CASSI system using a random coded aperture with a transmittance of 50%. Two reconstructed bands are presented

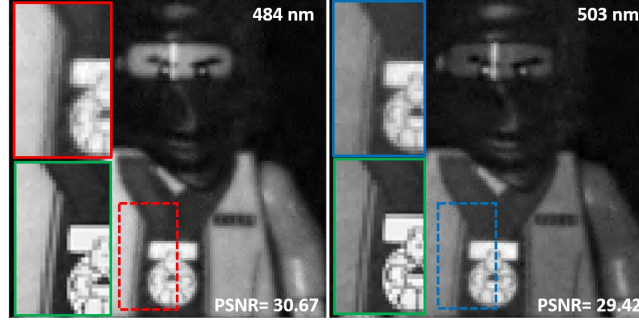


Figure 3. Two spectral bands reconstructed through simulation of the CASSI system and using a random coded aperture pattern. Zoomed sections are presented in red and blue squares. The original zoomed section is presented in a green square to visually compare the spatial quality of the reconstructions.

in figure 3, a zoom on the toy chest highlight the poor reconstruction of the edges, the same original section is presented in a green square in order to visually compare the quality.

The previous observations lead to propose the design of coded apertures based on the side information to enhance the reconstruction quality of the data cube. More formally, the  $k^{th}$  spectral band of the spatio-spectral image  $(\mathbf{F}_k)_{mn}$  defined before, for  $m, n = 0, \dots, N-1$  and  $k = 0, 1, \dots, L-1$  can be compactly rewritten as  $\mathbf{F}_k = (\mathbf{F}_k)_{mn}$ . In order to take advantage of the a-priori information, a RGB version  $\mathbf{F}_C = [\mathbf{F}_R \ \mathbf{F}_G \ \mathbf{F}_B]$  is used to design edge-based coded aperture patterns. Each RGB image channel can be written as

$$\mathbf{F}_R = \sum_{k=0}^{L-1} \mathbf{w}_k^R \mathbf{F}_k, \quad \mathbf{F}_G = \sum_{k=0}^{L-1} \mathbf{w}_k^G \mathbf{F}_k, \quad \mathbf{F}_B = \sum_{k=0}^{L-1} \mathbf{w}_k^B \mathbf{F}_k, \quad (3)$$

where  $\mathbf{w}_k^R, \mathbf{w}_k^G, \mathbf{w}_k^B > 0$ , give the spectral response of the CCD sensor for each of the three R,G,B channels. A filter to estimate the edges of the RGB image is applied following the Canny method [1]. For instance, for the R channel, the filtering is defined as,

$$h * \mathbf{F}_R = h * \left( \sum_{k=0}^{L-1} \mathbf{w}_k^R \mathbf{F}_k \right). \quad (4)$$

The edges of each channel are the linear combination of the edges of each band  $\mathbf{F}_k$ .

$$\hat{\mathbf{F}}_R = \sum_{k=0}^{L-1} \mathbf{w}_k^R (h * \mathbf{F}_k) \simeq \sum_{k=0}^{L-1} \mathbf{w}_k^R \hat{\mathbf{F}}_k, \quad (5)$$

The estimated edge image is then calculated as  $\hat{\mathbf{F}} = \hat{\mathbf{F}}_R + \hat{\mathbf{F}}_G + \hat{\mathbf{F}}_B$ . Then can be assumed that  $\hat{\mathbf{F}}$  contains information of all the spectral bands edges, and it is a good estimation of the edges for the spectral bands  $\hat{\mathbf{F}}_k$ .

The idea of the coded aperture design is to emphasize the edges. For this, we propose to design the codes as the hadamard product of two components, making a differentiation of the edges in the scene. The first component is a blue noise pattern generated by using a blue noise mask<sup>11</sup> in order to achieve a more uniform sensing.<sup>12</sup> The second component is the edge component,

$$\mathbf{T} = \mathbf{T}_{b1} \cdot \mathbf{T}_e + \mathbf{T}_{b2} \cdot (1 - \mathbf{T}_e), \quad (6)$$

where  $\mathbf{T}_{b1}$  and  $\mathbf{T}_{b2}$  are the blue noise patterns with a determined transmittance, and  $\mathbf{T}_e$  is the edge component. The precalculated image  $\hat{\mathbf{F}}$  is used as the edge component. Thus  $\mathbf{T}_e = \hat{\mathbf{F}}$ . Then the coded aperture in Eq. 6 can be re-written as,

$$\mathbf{T} = \mathbf{T}_{b1} \cdot \hat{\mathbf{F}} + \mathbf{T}_{b2} \cdot (1 - \hat{\mathbf{F}}). \quad (7)$$

The transmittance of the blue noise patterns  $\mathbf{T}_{b1}$  and  $\mathbf{T}_{b2}$  must be different, otherwise, the coded aperture can be seen as a blue noise pattern sensing the whole scene. Replacing the new coded aperture in Eq. 1, the FPA measurement just for  $\ell = 1$  snapshot can be re-written as,

$$Y_{mn} = \underbrace{\sum_{k=0}^{L-1} (\mathbf{T}_{b1})_{m(n-k)} (\hat{\mathbf{F}})_{m(n-k)} (\mathbf{F}_k)_{m(n-k)}}_a + \underbrace{\sum_{k=0}^{L-1} (\mathbf{T}_{b2})_{m(n-k)} (1 - \hat{\mathbf{F}})_{m(n-k)} (\mathbf{F}_k)_{m(n-k)}}_b, \quad (8)$$

where the first (a) part in Eq. 8 accounts for the sensing of the edge estimation for the different bands, and part (b) represents the sensing of the scene without the edge sections.

### 3.2 Reconstruction process

The CASSI with side information approach results in two shots of measurements. The CASSI measurements corresponding to Eq. 8, and the RGB shot  $\mathbf{F}_C$ . In order to reconstruct the spatio-spectral data cube, the two measurements are stacked together such that the final measurements are given by  $\tilde{\mathbf{Y}} = \tilde{\mathbf{H}}\mathbf{f} + \tilde{\boldsymbol{\omega}}$ , with

$$\tilde{\mathbf{Y}} = \begin{bmatrix} \mathbf{Y} \\ \mathbf{Y}' \end{bmatrix}, \quad \tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H}' \end{bmatrix}, \quad \tilde{\boldsymbol{\omega}} = \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{\omega}' \end{bmatrix}, \quad (9)$$

where  $\mathbf{Y} = \mathbf{H}\mathbf{f} + \boldsymbol{\omega}$  is the linear matrix representation of the CASSI shot, and  $\mathbf{Y}' = \mathbf{H}'\mathbf{f} + \boldsymbol{\omega}'$  corresponds to the linear representation of the RGB measurement.  $\mathbf{H}$  is the sensing matrix that represents the effects of the coded aperture and the dispersive element, and  $\mathbf{H}'$  is the system forward response of the RGB camera.  $\tilde{\boldsymbol{\omega}}$  accounts for the additive noise of the two detectors, generally modeled as white Gaussian noise.

The compressive sensing gradient projection for sparse reconstruction (GPSR) algorithm is used to obtain the reconstruction of the data cube.<sup>13</sup> This algorithm solves the optimization problem

$$\hat{\mathbf{f}} = \Psi \{ \operatorname{argmin}_{\boldsymbol{\theta}} \|\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\mathbf{f}\|_2 + \tau \|\boldsymbol{\theta}\|_1 \}, \quad (10)$$

where  $\boldsymbol{\theta}$  is an S-sparse representation of  $\mathbf{f}$ , and  $\tau$  is a regularization constant. The basis representation  $\Psi$  is set as the Kronecker product of two bases  $\Psi = \Psi_1 \otimes \Psi_2$ , where  $\Psi_1$  is a 2D wavelet Symmlet 8 basis and  $\Psi_2$  playing the role of spectral sparsifier is a the 1D discrete cosine transform.

## 4. SIMULATIONS AND EXPERIMENTAL RESULTS

### 4.1 Simulations

In order to verify the CASSI with side information approach, two set of compressive measurements are simulated. The first set uses a random coded aperture and the second set uses a designed coded aperture. A test spectral data cube  $\mathbf{F}$  was acquired using a monochromator in the spectral range between 450nm and 650nm with  $128 \times 128$  pixels of spatial resolution and  $L = 8$  spectral bands. A CCD camera AVT Marlin F0033B, with  $656 \times 492$  pixels and a pixel pitch size of  $9.9\mu\text{m}$  is used. The spatial resolution of the coded apertures is  $128 \times 128$  pixels, and the transmittances used is 0.25. Figure 4 depicts the random and designed coded apertures used and the spatial reconstruction of four spectral bands. It can be noticed that the designed coded aperture provides more detailed reconstructions and the quality achieved on the edges is noticeable. The spectral signatures of two different points in the scene are compared against a reference using a commercial spectrometer (Ocean Optics USB2000+) in Fig. 5. The curves obtained by using the designed coded apertures are closer to the original. The mean spectral PSNR for the eight reconstructed bands is also presented in Fig. 5 to evaluate the spectral reconstructions.



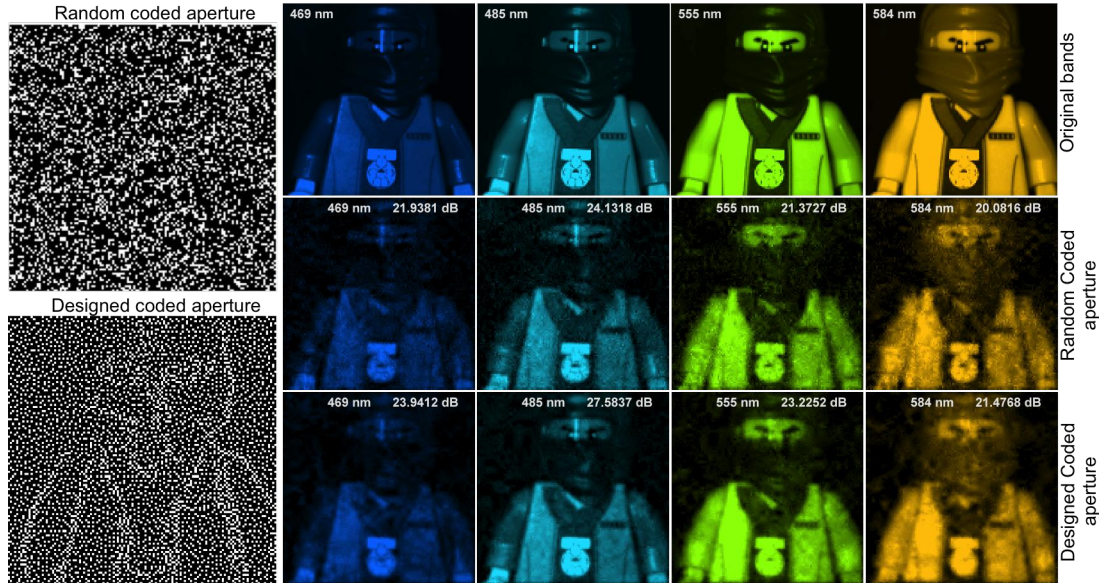


Figure 4. Reconstruction of four spectral bands using the CASSI with side information. Left column depicts the random and designed coded apertures used. Right columns depict the original bands, reconstruction using a random coded aperture and reconstruction using the designed coded aperture.

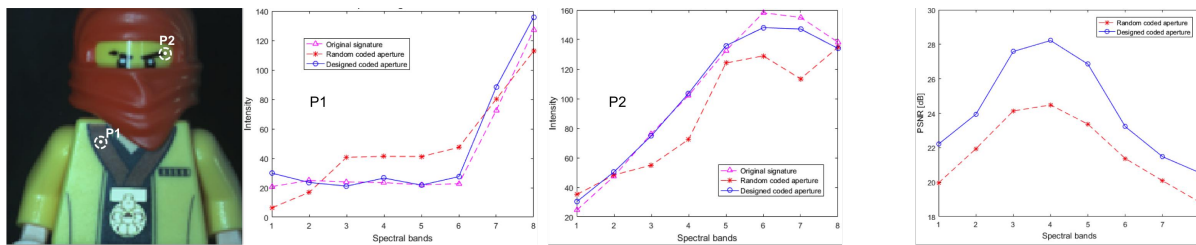


Figure 5. Spectral reconstructions. (Left) Spectral signatures for two representative spatial points indicated as P1 and P2. (Right) Mean spectral PSNR for the eight reconstructed bands.

## 4.2 Experimental setup and results

The optical setup of the CASSI system was constructed to experimentally demonstrate the performance of the proposed approach. For the setup, a CCD camera, a DMD, and a double Amici prism were used. The CCD camera has a resolution of  $656 \times 492$  pixels and a pixel pitch size of  $9.9\mu\text{m}$ . The DMD used to implement the coded apertures has a resolution of  $1024 \times 768$  pixels and a mirror pitch size of  $13.68\mu\text{m}$ . An RGB shot of the scene is captured to estimate the edges and to design the coded aperture pattern. Coded apertures with  $128 \times 128$  pixels are implemented in the DMD. As a result, the resolution of the final 2D projections and respective reconstructions is  $128 \times 128$  pixels with 10 spectral bands. Figure 6 illustrates the random and designed coded apertures used and the reconstruction of four spectral bands respectively. The quality obtained especially on the edges increases with the use of the designed coded aperture. Zommed sections show the quality of the edges obtained with the use of the designed coded aperture.

## 5. CONCLUSIONS

The use of side information in coded aperture compressive spectral imaging for the sensing and reconstruction process has been demonstrated. The design of coded aperture patterns based on the RGB side information leads to improved reconstructions as well as edge quality. The experimental results have shown improved reconstructions when the side information is employed in the system.

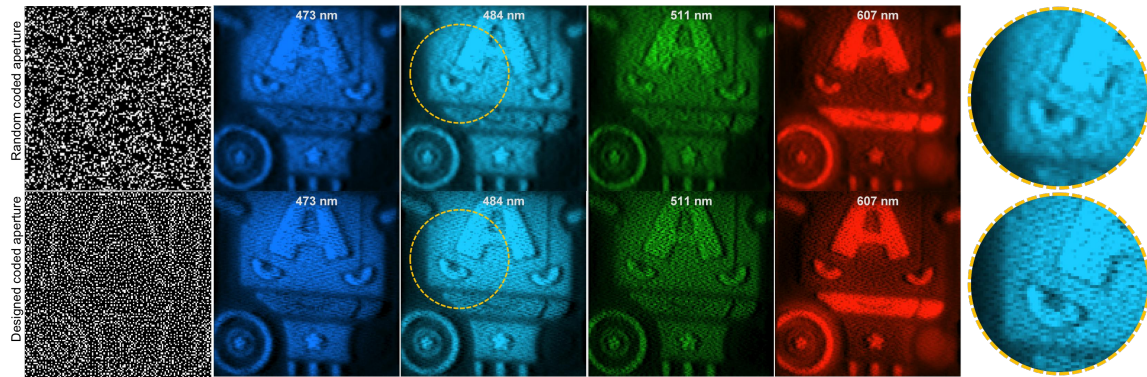


Figure 6. Experimental reconstruction of four spectral bands using the CASSI with side information. (Up) Random coded aperture. (Down) Designed coded aperture.

## ACKNOWLEDGMENTS

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